FULL RESPONSE TIME DISTRIBUTIONS IN ABSOLUTE IDENTIFICATION MODELED VIA LEAKY COMPETING DECISION PROCESSES

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Abstract

Lacouture and Marley have previously been quite successful in modeling the probability correct and the mean correct response time in unidimensional absolute identification tasks for various stimulus ranges and stimulus/response set sizes. These fits include those to a set of phenomenon often referred to as end-anchor effects. The present extension of Lacouture and Marley's mapping model is aimed at fitting not only probability correct and mean correct response time, but the whole distribution of (correct) response times as observed for the different elements of the stimulus set.

A typical absolute identification experiment involves a set of N stimuli that vary along some psychological dimension. On each trial, one of the N stimuli is presented, and the participant's task is to select the previously specified 'correct' response for that stimulus. The responses are usually key presses, with the keys being associated with the numerals 1 through N, with the usual order of the numerals being in agreement with the psychological 'magnitude' of the stimuli so identified. One of the major phenomena in this area is that no matter how widely spread the stimuli are on the relevant sensory dimension, people are usually only able to identify the stimuli up to an accuracy that corresponds to correctly identifying about seven stimuli. Although our own empirical and theoretical work includes study of this basic result, we have gone much further and studied a second set of phenomenon variously referred to as the *end-anchor effect,* the *bow effect,* and sometimes the *serial position effect* (Lacouture, 1997; Lacouture and Marley, 1991, 1995). These terms refer to the fact that accuracy decreases and response time increases as one moves away from the stimuli at the end of the presented range; the effects also become more pronounced for larger set sizes (larger N).

Previously we have successfully modeled the relevant accuracy data and the general pattern of the mean correct response time data (Lacouture and Marley, 1998). However, until now we have not applied our model to complete response time distributions. The revised model presented here is directed at eliminating these limitations of our prior work.

Both our original and revised *mapping model* of unidimensional absolute identification consists of a three-layer connectionist feed-forward network with linear units. The sensory input is mapped to a (bounded) unidimensional (scalar) internal representation, which is in turn mapped via a theoretically motivated set of linear functions to an N-dimensional output vector. A key feature of the model is the specific set of weights and biases used in the linear mappings of the internal scalar representation to the N-dimensional output vector. This output vector in turn provides the input to the decision process. The addition of Gaussian noise at each of the input, internal, and output level of the process enables the mapping model, with a single set of parameter values, to reproduce core characteristics of identification and categorization data. The combination of our basic *mapping model* with a *leaky competing accumulator decision process*

(Usher and McClelland, 2001) allows us to model full RT distributions. Lacouture and Marley (2000) presented theoretical results. The current paper presents comparison of empirical and simulation results.

The leaky competing decision process

For an absolute identification task with N stimuli, the simulated model involves N decision units (*accumulators*). Using Usher and McClelland's (2001) notation, let x_i be the activation signal received by accumulator i, and let f_i be the firing rate of accumulator *i* such that $f_i = x_i$ for $x_i > 0$ and $f_i = 0$ for $x_i < 0$. We assume that a response is made when a single accumulator remains active and has reached an equilibrium state. Response time is the time taken for the network to reach equilibrium. **(**For now, we are implementing 'reaching equilibrium' by waiting until only one unit is active, and its activity has changed less than a chosen amount over a number of trials). For each accumulator the change in activation follows

$$dx_i = \left[\rho_i - \lambda x_i + \alpha f_i - \beta \sum_{i' \neq i} f_{i'}\right]\frac{dt}{\tau} + \xi_i \sqrt{\frac{dt}{\tau}} \quad , \qquad (1)$$

where the change in activation for accumulator i depends on: the external input with value ρ_i - in the present case, this value is given by the output from the mapping model; the leakage parameter λ; the self amplification parameter α; and the inhibitory signal from the other accumulators with parameter β. For $x_i > 0$, the equation is linear and we can replace the value $(\lambda - \alpha)$ by a net leakage parameter k. For $k > 0$ activations decay, and for $k < 0$ activations self-amplify. We can replace the firing rates f_i in Eq. (1) by activation levels x_i, and truncate the x_i at zero. We then have

$$\begin{aligned} dx_i &= \left[\rho_i - kx_i - \beta \sum_{i' \neq i} x_{i'}\right]\frac{dt}{\tau} + \xi_i \sqrt{\frac{dt}{\tau}} \\ x_i &\to \max(x_i, 0). \end{aligned} \qquad (2)$$

Simulations by Usher & McClelland (2001) show that Eq. (2) gives a close approximation to Eq. (1) Because of the inhibitory processes when $k > 0$, the activations converge towards zero for all but a single unit. Also, the asymptotic activation in that remaining unit reaches an equilibrium level where the net input to the accumulator equals the sum of the inhibitory and leakage signals. Again, the 'overt' response associated with the 'winning' unit is made when an equilibrium state is reached - see details presented just before Eq. (1).

Empirical experiment

To test how the revised model can fit RT distributions, data were collected from a single participant who was required to perform 10 sessions of an absolute identification task involving 10 line lengths. The details of the task are presented in Lacouture (1997). Each session had 300 trials, which provides a good representation of the RT distribution (for correct responses) for each stimulus.

Method

Apparatus and General Procedure

The apparatus and general procedure are the same as were used in Lacouture (1997). The participant performed identification of visual stimuli. Responses were collected using a custom-made keyboard with 11 buttons. On this keyboard, one button, labeled "START", is located at the center of the ten other buttons positioned in a semi-circle such that the distance between the START button and each of the other response buttons is equal (101 mm). Each response button corresponds to one of the possible stimuli and the button arrangement corresponds to the natural ordering of the stimuli from the smallest (leftmost key) to the largest (rightmost key). The participant initiated each trial by pressing the START button. One randomly selected stimulus was shown on the screen 100 ms later. The participant had to identify the stimulus by pressing one of the appropriate response buttons. Feedback was provided for one second in the form of a number corresponding to the ordinal position of the stimulus. If the participant made an incorrect response, a low frequency (500 Hz) tone was generated for 500 ms.

Participant

One graduate student performed a total of 10 experimental sessions each consisting of 300 trials.

Stimuli

The stimulus set consisted of ten lines of different lengths; on each trial, one of these 10 stimuli was presented horizontally in the middle of the computer screen. From the participant's viewpoint the stimuli appeared like continuous lines. The length of each segment, measured in pixel units (screen dots), was 92, 106, 120, 138, 160, 184, 212, 242, 278, and 320, respectively. Within the stimulus set, each successive stimulus was 15 % longer than the previous one. Consequently, each adjacent pair of stimuli was well above Weber's fraction for line length. The stimuli were given "correct" response labels of “1” to “10” according to increasing length.

Results

Trials associated with extreme response time values (2 % at each extreme of the percentile distribution for each stimulus) were removed. Only correct trials were used for response time results. The right panels of Figure 1 report probability correct (PC), mean correct response time (MRT), and standard deviation of RTs, plotted according to stimulus position.

Analysis of response time distributions

The distribution of correct response times for each stimulus in each condition was analyzed using maximum likelihood estimations. The best fitting parameters were found for the ex-Gaussian, Gaussian and Weibull functions. Using the AIC criterion, the ex-Gaussian proved in all cases to provide a superior fit to the empirical distributions. The ex-Gaussian function is the convolution of a Gaussian (normal) function with an exponential function and is written as:

$$\mathrm{f}(t) = \frac{1}{\tau}\exp\left(\frac{\mu}{\tau} + \frac{\sigma^2}{2\tau^2} - \frac{t}{\tau}\right)\Phi\left(\frac{t-\mu-\sigma^2/\tau}{\sigma}\right),\ t > 0. \quad (3)$$

In this equation, the exponential function is multiplied by the value of the cumulative density function of the Gaussian function symbolized by Φ. The resulting ex-Gaussian function has three parameters, μ, σ, and τ. The two first parameters (μ and σ) correspond to the mean and standard deviation of the Gaussian component. The third parameter (τ) is the mean of the exponential component.

The bottom panels of Figure 2 present, for each stimulus, a histogram representing the distribution of RT with the overlayed best fitting ex-Gaussian function.

Simulating the absolute identification task

In order to replicate the empirical data, a simulation of the mapping model was run using the leaky competing accumulator decision process. The output from the mapping process became the input to the leaky competing decision process. We obtained a good fit of the simulated data to the empirical data with the following parameters: input noise $\gamma = 0.35$, hidden noise $\eta = 0.02$, output noise $\varepsilon = 0.02$ and, for the leaky competing parameters, $\beta = 0.25$ and $k = 0.1$. Figure 1 illustrates the overall correspondence between the empirical (right panels) and simulated (left panels) results. Note that the simulated response times were converted to ms using a regression function of the simulated RTs (in network steps) on the empirical RTs (in ms). In all cases the results are plotted according to stimulus position. Overall, the results of the simulated process closely matches the empirical data.

<u>Response time distributions of the simulated process</u>

Best fitting parameters for the Gaussian, ex-Gaussian and Weibull functions were estimated for each of the ten distributions (each of the stimuli). Figure 2 presents the simulated distributions overlayed with the best fitting ex-Gaussian function. For both the empirical and simulated data, the distributions are best represented by ex-Gaussian functions. Also, in both cases, the shape of the distributions approach an Exponential function towards the ends of the stimulus continuum (parameter σ tends toward zero). For all stimuli, a similar distribution is observed for the empirical and simulated results. Figure 3 reports the estimated parameters of the ex-Gaussian function plotted according to stimulus position for both the simulated and the empirical results. The top left panel presents the estimated μ's, the top right panel the estimated σ's, and the bottom panel the estimated τ's. The figure shows very good fit for the parameters μ and τ and a possibly significant discrepancy between simulated and empirical results for parameter σ. We must complete further simulations, with variations in σ, to understand how sensitive the fits are to changes in that parameter.

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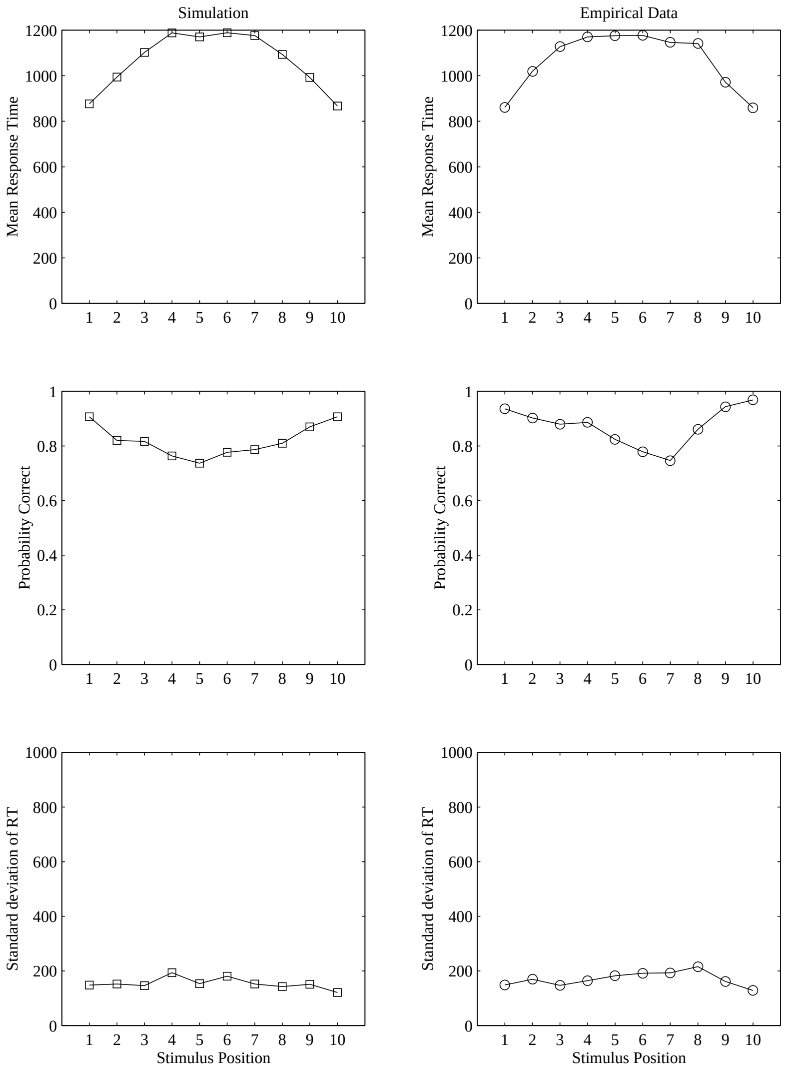

Figure 1. Results of the absolute identification experiment. Mean correct response time, probability correct, and standard deviation of RTs, plotted according to stimulus position for empirical (right panels) and simulated (left panels) data.

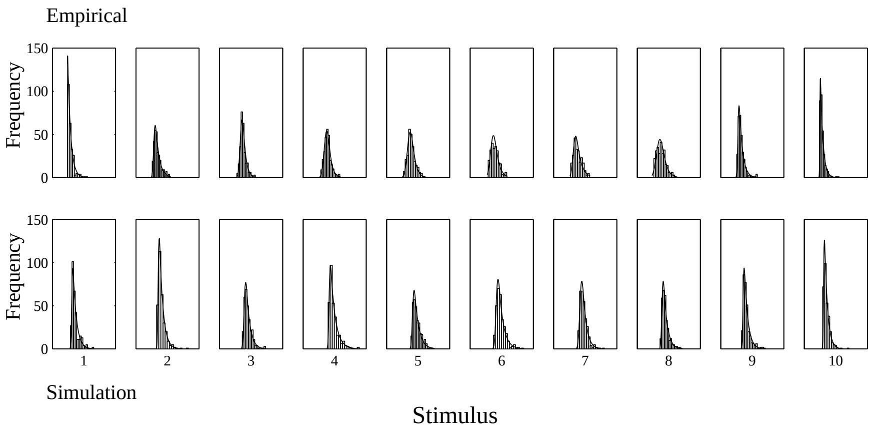

Figure 2. Observed RT distribution plotted for each stimulus with the overlayed best fitting ex-Gaussian function. Top row: empirical data, bottom row: simulation results.

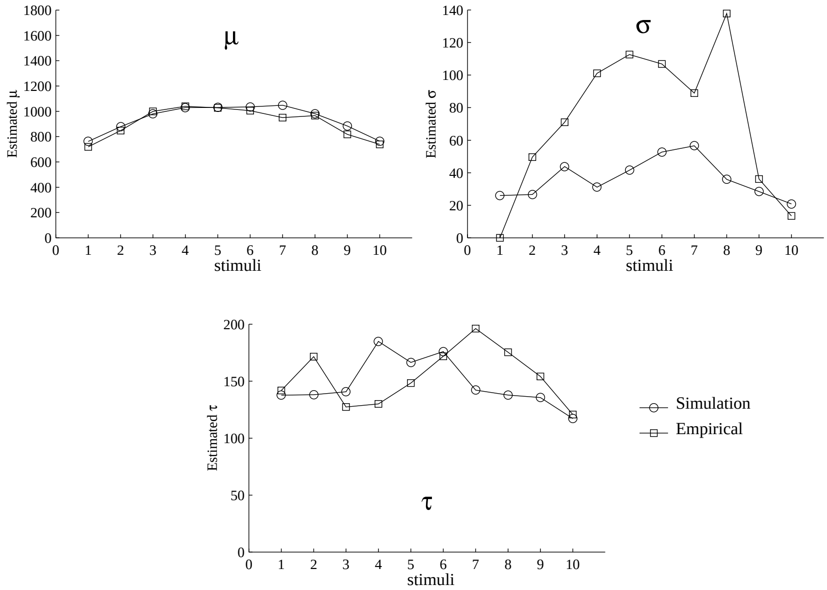

Figure 3. Estimated parameters of the ex-Gaussian function plotted according to stimulus position for both the simulated and the empirical results.